

**MATH4250 Game Theory**  
**Exercise 3**

Assignment 3: 1(b), 2(a)(c)(f) (Due: 27 Feb 2019 (Wednesday))

1. Solve the following primal problems. Then write down the dual problems and the solutions to the dual problems.

(a)

$$\begin{aligned} \max \quad & f = 3y_1 + 5y_2 + 4y_3 + 12 \\ \text{subject to} \quad & 3y_1 + 2y_2 + 2y_3 \leq 15 \\ & 4y_2 + 5y_3 \leq 24 \end{aligned}$$

(b)

$$\begin{aligned} \max \quad & f = 2y_1 + 4y_2 + 3y_3 + y_4 \\ \text{subject to} \quad & 3y_1 + y_2 + y_3 + 4y_4 \leq 12 \\ & y_1 - 3y_2 + 2y_3 + 3y_4 \leq 7 \\ & 2y_1 + y_2 + 3y_3 - y_4 \leq 10 \end{aligned}$$

2. Solve the zero sum games with the following game matrices, that is find the value of the game, a maximin strategy for the row player and a minimax strategy for the column player.

(a)  $\begin{pmatrix} 2 & -3 & 3 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{pmatrix}$

(d)  $\begin{pmatrix} 2 & 0 & -2 \\ -1 & -3 & 3 \\ -2 & 2 & 0 \end{pmatrix}$

(b)  $\begin{pmatrix} 3 & 1 & -5 \\ -1 & -2 & 6 \\ -2 & -1 & 3 \end{pmatrix}$

(e)  $\begin{pmatrix} 1 & -1 & 1 \\ -2 & 0 & -1 \\ 1 & -2 & 2 \\ -1 & 1 & -2 \end{pmatrix}$

(c)  $\begin{pmatrix} 3 & 0 & 1 \\ -1 & 2 & -2 \\ 0 & 1 & -1 \end{pmatrix}$

(f)  $\begin{pmatrix} -3 & 2 & 0 \\ 1 & -2 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & -3 \end{pmatrix}$

3. Prove that if  $C_1$  and  $C_2$  are convex sets in  $\mathbb{R}^n$ , then the following sets are also convex.

(a)  $C_1 \cap C_2$

(b)  $C_1 + C_2 = \{\mathbf{x}_1 + \mathbf{x}_2 : \mathbf{x}_1 \in C_1, \mathbf{x}_2 \in C_2\}$

4. Let  $A$  be an  $m \times n$  matrix. Prove that the set of maximin strategies for the row player of  $A$  is convex.

5. Let  $C$  be a convex set in  $\mathbb{R}^n$  and  $\mathbf{x}, \mathbf{y} \in C$ . Let  $\mathbf{z} \in \mathbb{R}^n$  be a point on the straight line joining  $\mathbf{x}$  and  $\mathbf{y}$  such that  $\mathbf{z}$  is orthogonal to  $\mathbf{x} - \mathbf{y}$ .

(a) Find  $\mathbf{z}$  in terms of  $\mathbf{x}$  and  $\mathbf{y}$ .

(b) Suppose  $\langle \mathbf{x}, \mathbf{y} \rangle < 0$ . Prove that  $\mathbf{z} \in C$ .

6. Let  $A$  be an  $m \times n$  matrix with column vectors  $\mathbf{a}_1^T, \mathbf{a}_2^T, \dots, \mathbf{a}_n^T$ . Let  $\nu_c(A)$  be the column value of  $A$  and let

$$C = \text{Conv}(\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n, \mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_m\})$$

where  $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_m\}$  is the standard basis for  $\mathbb{R}^m$ . Prove that if  $\nu_c(A) \leq 0$ , then  $\mathbf{0} \in C$ .